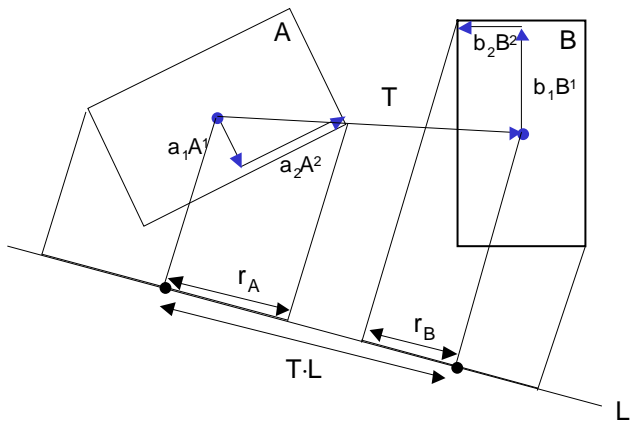


Fast Overlap Test for OBBs

2002.9.4

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(convex polygon¹) (project),
 (overlap) (separating axis),
 (edge) (polygon) (face)
 (orthogonal) (axis) (plane)
 1
 OBB(Oriented Bounding Box) 가
 (potential separating axis) 1
 (), OBB



1. $OBB(A \cap B)$ (L) - $|L|=1$ 가

OBB (separating axis) 가 OBB OBB
 / (face/face), / (face/edge), / (face-vertex), / (edge/edge),
 / (edge/vertex), / (vertex/vertex) OBB
 (plane) (face-face,

¹ (Convex polygon) polygon
 (segment) (polygon)

face-edge, face-vertex), OBB (separating axis) (normal) .(box face가 , 6 가) edge-edge 가 2 .(OBB edge box edge 3 3x3 9 가 .) edge-vertex 가 vertex-vertex box 가 . object A A¹, A², A³ object B B¹, B², B³ 가 . OBB가 , A B . object A B¹, B², B³ . , A¹, A², A³ 가 . A B , . 6 : A¹, A², A³, B¹, B², B³ 9 : A¹×B¹, A¹×B², A¹×B³, A²×B¹, A²×B², A²×B³, A³×B¹, A³×B², A³×B³ (B¹, B², B³ A , B¹, B², B³ . B¹, B², B³ B .)

, OBB B OBB A . OBB A object A world

$$A|_{world} = T_A R_A A|_{objectA}$$

OBB B object B world

$$B|_{world} = T_B R_B A|_{objectB}$$

, object B object A

$$B|_{objectA} = (T_A R_A)^{-1} T_B R_B B|_{objectB} .$$

, vector B , r_A r_B . object B B

, r_A, r_B R

$$R = R_A^{-1} R_B$$

2 가 가 .

OBB
 A_1, A_2, A_3 , object B
 $2a_1, 2a_2, 2a_3$, OBB B

vector
 B_1, B_2, B_3
 가

T
 , object A
 , OBB A
 $2b_1, 2b_2, 2b_3$
 가

$$r_A = \frac{|a_1 A_1 \cdot L|}{|L|} + \frac{|a_2 A_2 \cdot L|}{|L|} + \frac{|a_3 A_3 \cdot L|}{|L|}$$

$$r_B = \frac{|b_1 RB_1 \cdot L|}{|L|} + \frac{|b_2 RB_2 \cdot L|}{|L|} + \frac{|b_3 RB_3 \cdot L|}{|L|}$$

$$\frac{|T \cdot L|}{|L|} < r_A + r_B \quad L \quad \text{OBB}$$

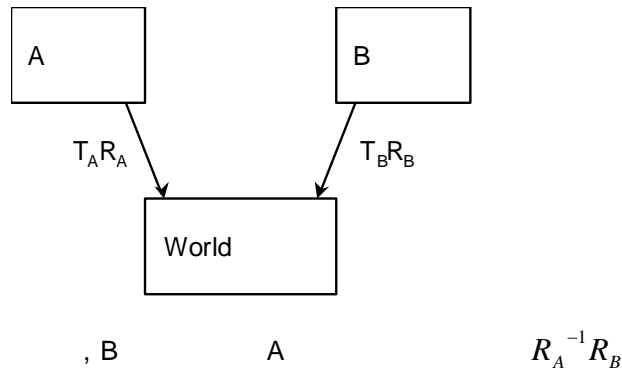
Appendix OBBs
 . Appendix 가 15 가
 15 가
 , 15 가 가 ,
 OBB

1. S.Gottshcalk, M. C. Lin, D. Manocha, OBBTree : A Hierarchical Structure for Rapid Interface Detection, <http://www.cs.unc.edu/~geom/OBB/OBBT.html>
2. , Collision Detection, 3D Game Engine <http://www.minpress.com/gna/files/kbg.pdf>

³ 1 $|L|=1$ 가 , $|L|=1$ 가
 OBB 가 , cross product
 가 1 $|L|$ 가 1 가

Appendix

- i) A A
- ii) B A
- iii) A world, () : T_A, R_A
- iv) B world, () : T_B, R_B
- v) , 1 $r_A r_B$



- vi) T A OBB A OBB B
vector .
, $T = [\text{center of B}]_A - [\text{center of A}]_A = [\text{center of B}]_A = R_A^{-1} R_B [0 \ 0 \ 0]^T$
- vii) A^i A , A normal vector

$$A^1 = [1 \ 0 \ 0]$$

$$A^2 = [0 \ 1 \ 0]$$

$$A^3 = [0 \ 0 \ 1]$$

- viii) T R
 $T = [T_1 \ T_2 \ T_3]$

$$R = R_A^{-1} R_B = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}$$

$$RB^1 = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_{11} \\ R_{21} \\ R_{31} \end{bmatrix}$$

$$RB^2 = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} R_{12} \\ R_{22} \\ R_{32} \end{bmatrix}$$

$$RB^3 = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} R_{13} \\ R_{23} \\ R_{33} \end{bmatrix}$$

$$r_A = \frac{|a_1 A^1 \cdot L|}{|L|} + \frac{|a_2 A^2 \cdot L|}{|L|} + \frac{|a_3 A^3 \cdot L|}{|L|}, \quad r_B = \frac{|b_1 RB^1 \cdot L|}{|L|} + \frac{|b_2 RB^2 \cdot L|}{|L|} + \frac{|b_3 RB^3 \cdot L|}{|L|}$$

$$\frac{|T \cdot L|}{|L|}$$

$$\frac{|T \cdot L|}{|L|} > r_A + r_B, \quad |L|$$

|L|

$$|T \cdot L| > (r_A + r_B)|L| = |a_1 A^1 \cdot L| + |a_2 A^2 \cdot L| + |a_3 A^3 \cdot L| + |b_1 RB^1 \cdot L| + |b_2 RB^2 \cdot L| + |b_3 RB^3 \cdot L|$$

$$|T \cdot L| > (r_A + r_B)|L|, \quad L \text{ separating axis} \quad A \quad B$$

OBB L

15

1) $L = A^1$

$$r_A |L| = |a_1 A^1 \cdot A^1| + |a_2 A^2 \cdot A^1| + |a_3 A^3 \cdot A^1| = a_1$$

$$r_B |L| = |b_1 RB^1 \cdot A^1| + |b_2 RB^2 \cdot A^1| + |b_3 RB^3 \cdot A^1| = b_1 |R_{11}| + b_2 |R_{12}| + b_3 |R_{13}|$$

$$|T \cdot L| = |T \cdot A^1| = \begin{bmatrix} T_1 & T_2 & T_3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = |T_1|$$

$$|T_1| > a_1 + b_1 |R_{11}| + b_2 |R_{12}| + b_3 |R_{13}| \quad A^1 \text{ separating axis가}$$

$$2) L = A^2$$

$$r_A |L| = |a_1 A^1 \cdot A^2| + |a_2 A^2 \cdot A^2| + |a_3 A^3 \cdot A^2| = a_2$$

$$r_B |L| = |b_1 RB^1 \cdot A^2| + |b_2 RB^2 \cdot A^2| + |b_3 RB^3 \cdot A^2| = b_1 |R_{21}| + b_2 |R_{22}| + b_3 |R_{23}|$$

$$|T \cdot L| = |T \cdot A^2| = \begin{bmatrix} T_1 & T_2 & T_3 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} = |T_2|$$

$$|T_2| > a_2 + b_1 |R_{21}| + b_2 |R_{22}| + b_3 |R_{23}| \quad A^2 \quad \text{separating axis가}$$

$$3) L = A^3$$

$$r_A |L| = |a_1 A^1 \cdot A^3| + |a_2 A^2 \cdot A^3| + |a_3 A^3 \cdot A^3| = a_3$$

$$r_B |L| = |b_1 RB^1 \cdot A^3| + |b_2 RB^2 \cdot A^3| + |b_3 RB^3 \cdot A^3| = b_1 |R_{31}| + b_2 |R_{32}| + b_3 |R_{33}|$$

$$|T \cdot L| = |T \cdot A^3| = \begin{bmatrix} T_1 & T_2 & T_3 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} = |T_3|$$

$$|T_3| > a_3 + b_1 |R_{31}| + b_2 |R_{32}| + b_3 |R_{33}| \quad A^3 \quad \text{separating axis가}$$

$$4) L = RB^1 = \begin{bmatrix} R_{11} & R_{21} & R_{31} \end{bmatrix}$$

$$r_A |L| = |a_1 A^1 \cdot RB^1| + |a_2 A^2 \cdot RB^1| + |a_3 A^3 \cdot RB^1| = |a_1 R_{11}| + |a_2 R_{21}| + |a_3 R_{31}|$$

$$r_B |L| = |b_1 RB^1 \cdot RB^1| + |b_2 RB^2 \cdot RB^1| + |b_3 RB^3 \cdot RB^1| = |b_1| = b_1$$

$$\begin{matrix} B^1, B^2, B^3 & A & RB^1, RB^2, RB^3 \\ \cdot & RB^1 \cdot RB^2 = 0, RB^1 \cdot RB^3 = 0, RB^2 \cdot RB^3 = 0 & \cdot \end{matrix}$$

$$|T \cdot L| = |T \cdot RB^1| = \begin{bmatrix} T_1 & T_2 & T_3 \end{bmatrix} \cdot \begin{bmatrix} R_{11} & R_{21} & R_{31} \end{bmatrix} = |T_1 R_{11} + T_2 R_{21} + T_3 R_{31}|$$

$$|T_1 R_{11} + T_2 R_{21} + T_3 R_{31}| > a_1 |R_{11}| + a_2 |R_{21}| + a_3 |R_{31}| + b_1 \quad RB^1 \quad \text{separating axis가}$$

$$5) L = RB^2 = \begin{bmatrix} R_{12} & R_{22} & R_{32} \end{bmatrix}$$

$$r_A |L| = |a_1 A^1 \cdot RB^2| + |a_2 A^2 \cdot RB^2| + |a_3 A^3 \cdot RB^2| = |a_1 R_{12}| + |a_2 R_{22}| + |a_3 R_{32}|$$

$$r_B |L| = |b_1 RB^1 \cdot RB^2| + |b_2 RB^2 \cdot RB^2| + |b_3 RB^3 \cdot RB^2| = |b_2| = b_2$$

$$|T \cdot L| = |T \cdot RB^2| = [T_1 \quad T_2 \quad T_3] \cdot [R_{12} \quad R_{22} \quad R_{32}] = |T_1 R_{12} + T_2 R_{22} + T_3 R_{32}|$$

$$|T_1 R_{12} + T_2 R_{22} + T_3 R_{32}| > a_1 |R_{12}| + a_2 |R_{22}| + a_3 |R_{32}| + b_2 \quad RB^2 \quad \text{separating axis}$$

$$6) L = RB^3 = [R_{13} \quad R_{23} \quad R_{33}]$$

$$r_A |L| = |a_1 A^1 \cdot RB^3| + |a_2 A^2 \cdot RB^3| + |a_3 A^3 \cdot RB^3| = |a_1 R_{13}| + |a_2 R_{23}| + |a_3 R_{33}|$$

$$r_B |L| = |b_1 RB^1 \cdot RB^3| + |b_2 RB^2 \cdot RB^3| + |b_3 RB^3 \cdot RB^3| = |b_3| = b_3$$

$$|T \cdot L| = |T \cdot RB^3| = [T_1 \quad T_2 \quad T_3] \cdot [R_{13} \quad R_{23} \quad R_{33}] = |T_1 R_{13} + T_2 R_{23} + T_3 R_{33}|$$

$$|T_1 R_{13} + T_2 R_{23} + T_3 R_{33}| > a_1 |R_{13}| + a_2 |R_{23}| + a_3 |R_{33}| + b_3 \quad RB^3 \quad \text{separating axis}$$

$$7) L = A^1 \times RB^1 = [1 \quad 0 \quad 0] \times [R_{11} \quad R_{21} \quad R_{31}] = [0 \quad -R_{31} \quad R_{21}]$$

$$r_A |L| = |a_1 A^1 \cdot A^1 \times RB^1| + |a_2 A^2 \cdot A^1 \times RB^1| + |a_3 A^3 \cdot A^1 \times RB^1|$$

$$= |a_1 RB^1 \cdot A^1 \times A^1| + |a_2 RB^1 \cdot A^2 \times A^1| + |a_3 RB^1 \cdot A^3 \times A^1|$$

$$= |a_1 RB^1 \cdot 0| + |a_2 RB^1 \cdot (-A^3)| + |a_3 RB^1 \cdot A^2|$$

$$= |-a_2 R_{31}| + |a_3 R_{21}| = a_2 |R_{31}| + a_3 |R_{21}|$$

$$r_B |L| = |b_1 RB^1 \cdot A^1 \times RB^1| + |b_2 RB^2 \cdot A^1 \times RB^1| + |b_3 RB^3 \cdot A^1 \times RB^1|$$

$$= |b_1 A^1 \cdot RB^1 \times RB^1| + |b_2 A^1 \cdot RB^1 \times RB^2| + |b_3 A^1 \cdot RB^1 \times RB^3|$$

$$= |b_1 A^1 \cdot 0| + |b_2 A^1 \cdot RB^3| + |b_3 A^1 \cdot (-RB^2)|$$

$$= |b_2 R_{13}| + |-b_3 R_{12}| = b_2 |R_{13}| + b_3 |R_{12}|$$

$$|T \cdot L| = |T \cdot A^1 \times RB^1| = [T_1 \quad T_2 \quad T_3] \cdot [0 \quad -R_{31} \quad R_{21}] = |-T_2 R_{31} + T_3 R_{21}|$$

$$|-T_2 R_{31} + T_3 R_{21}| > a_2 |R_{31}| + a_3 |R_{21}| + b_2 |R_{13}| + b_3 |R_{12}| \quad A^1 \times RB^1 \quad \text{separating axis}$$

$$8) L = A^1 \times RB^2 = [1 \ 0 \ 0] \times [R_{12} \ R_{22} \ R_{32}] = [0 \ -R_{32} \ R_{22}]$$

$$\begin{aligned} r_A |L| &= |a_1 A^1 \cdot A^1 \times RB^2| + |a_2 A^2 \cdot A^1 \times RB^2| + |a_3 A^3 \cdot A^1 \times RB^2| \\ &= |a_1 RB^2 \cdot A^1 \times A^1| + |a_2 RB^2 \cdot A^2 \times A^1| + |a_3 RB^2 \cdot A^3 \times A^1| \\ &= |a_1 RB^2 \cdot 0| + |a_2 RB^2 \cdot (-A^3)| + |a_3 RB^2 \cdot A^2| \\ &= |-a_2 R_{32}| + |a_3 R_{22}| = a_2 |R_{32}| + a_3 |R_{22}| \end{aligned}$$

$$\begin{aligned} r_B |L| &= |b_1 RB^1 \cdot A^1 \times RB^2| + |b_2 RB^2 \cdot A^1 \times RB^2| + |b_3 RB^3 \cdot A^1 \times RB^2| \\ &= |b_1 A^1 \cdot RB^2 \times RB^1| + |b_2 A^1 \cdot RB^2 \times RB^2| + |b_3 A^1 \cdot RB^2 \times RB^3| \\ &= |b_1 A^1 \cdot (-RB^3)| + |b_2 A^1 \cdot 0| + |b_3 A^1 \cdot RB^1| \\ &= |-b_1 R_{13}| + |b_3 R_{11}| = b_1 |R_{13}| + b_3 |R_{11}| \end{aligned}$$

$$|T \cdot L| = |T \cdot A^1 \times RB^2| = \begin{vmatrix} T_1 & T_2 & T_3 \\ 0 & -R_{32} & R_{22} \end{vmatrix} = |-T_2 R_{32} + T_3 R_{22}|$$

$$|-T_2 R_{32} + T_3 R_{22}| > a_2 |R_{32}| + a_3 |R_{22}| + b_1 |R_{13}| + b_3 |R_{11}| \quad A^1 \times RB^2 \quad \text{separating axis}$$

$$9) L = A^1 \times RB^3 = [1 \ 0 \ 0] \times [R_{13} \ R_{23} \ R_{33}] = [0 \ -R_{33} \ R_{23}]$$

$$\begin{aligned} r_A |L| &= |a_1 A^1 \cdot A^1 \times RB^3| + |a_2 A^2 \cdot A^1 \times RB^3| + |a_3 A^3 \cdot A^1 \times RB^3| \\ &= |a_1 RB^3 \cdot A^1 \times A^1| + |a_2 RB^3 \cdot A^2 \times A^1| + |a_3 RB^3 \cdot A^3 \times A^1| \\ &= |a_1 RB^3 \cdot 0| + |a_2 RB^3 \cdot (-A^3)| + |a_3 RB^3 \cdot A^2| \\ &= |-a_2 R_{33}| + |a_3 R_{23}| = a_2 |R_{33}| + a_3 |R_{23}| \end{aligned}$$

$$\begin{aligned} r_B |L| &= |b_1 RB^1 \cdot A^1 \times RB^3| + |b_2 RB^2 \cdot A^1 \times RB^3| + |b_3 RB^3 \cdot A^1 \times RB^3| \\ &= |b_1 A^1 \cdot RB^3 \times RB^1| + |b_2 A^1 \cdot RB^3 \times RB^2| + |b_3 A^1 \cdot RB^3 \times RB^3| \\ &= |b_1 A^1 \cdot RB^2| + |-b_2 A^1 \cdot RB^1| + |b_3 A^1 \cdot 0| \\ &= |b_1 R_{12}| + |-b_2 R_{11}| = b_1 |R_{12}| + b_2 |R_{11}| \end{aligned}$$

$$|T \cdot L| = |T \cdot A^1 \times RB^3| = \begin{vmatrix} T_1 & T_2 & T_3 \\ 0 & -R_{33} & R_{23} \end{vmatrix} = |-T_2 R_{33} + T_3 R_{23}|$$

$$|-T_2 R_{33} + T_3 R_{23}| > a_2 |R_{33}| + a_3 |R_{23}| + b_1 |R_{12}| + b_2 |R_{11}| \quad A^1 \times RB^3 \quad \text{separating axis}$$

$$10) L = A^2 \times RB^1 = [0 \ 1 \ 0] \times [R_{11} \ R_{21} \ R_{31}] = [R_{31} \ 0 \ -R_{11}]$$

$$\begin{aligned} r_A |L| &= |a_1 A^1 \cdot A^2 \times RB^1| + |a_2 A^2 \cdot A^2 \times RB^1| + |a_3 A^3 \cdot A^2 \times RB^1| \\ &= |a_1 RB^1 \cdot A^1 \times A^2| + |a_2 RB^1 \cdot A^2 \times A^2| + |a_3 RB^1 \cdot A^3 \times A^2| \\ &= |a_1 RB^1 \cdot A^3| + |a_2 RB^1 \cdot 0| + |a_3 RB^1 \cdot (-A^1)| \\ &= |a_1 R_{31}| + |-a_3 R_{11}| = a_1 |R_{31}| + a_3 |R_{11}| \end{aligned}$$

$$\begin{aligned} r_B |L| &= |b_1 RB^1 \cdot A^2 \times RB^1| + |b_2 RB^2 \cdot A^2 \times RB^1| + |b_3 RB^3 \cdot A^2 \times RB^1| \\ &= |b_1 A^2 \cdot RB^1 \times RB^1| + |b_2 A^2 \cdot RB^1 \times RB^2| + |b_3 A^2 \cdot RB^1 \times RB^3| \\ &= |b_1 A^2 \cdot 0| + |b_2 A^2 \cdot RB^3| + |-b_3 A^2 \cdot RB^2| \\ &= |b_2 R_{23}| + |-b_3 R_{22}| = b_2 |R_{23}| + b_3 |R_{22}| \end{aligned}$$

$$|T \cdot L| = |T \cdot A^2 \times RB^1| = |[T_1 \ T_2 \ T_3] \cdot [R_{31} \ 0 \ -R_{11}]] = |T_1 R_{31} - T_3 R_{11}|$$

$$|T_1 R_{31} - T_3 R_{11}| > a_1 |R_{31}| + a_3 |R_{11}| + b_2 |R_{23}| + b_3 |R_{22}| \quad A^2 \times RB^1 \quad \text{separating axis}$$

$$11) L = A^2 \times RB^2 = [0 \ 1 \ 0] \times [R_{12} \ R_{22} \ R_{32}] = [R_{32} \ 0 \ -R_{12}]$$

$$\begin{aligned} r_A |L| &= |a_1 A^1 \cdot A^2 \times RB^2| + |a_2 A^2 \cdot A^2 \times RB^2| + |a_3 A^3 \cdot A^2 \times RB^2| \\ &= |a_1 RB^2 \cdot A^1 \times A^2| + |a_2 RB^2 \cdot A^2 \times A^2| + |a_3 RB^2 \cdot A^3 \times A^2| \\ &= |a_1 RB^2 \cdot A^3| + |a_2 RB^2 \cdot 0| + |a_3 RB^2 \cdot (-A^1)| \\ &= |a_1 R_{32}| + |-a_3 R_{12}| = a_1 |R_{32}| + a_3 |R_{12}| \end{aligned}$$

$$\begin{aligned} r_B |L| &= |b_1 RB^1 \cdot A^2 \times RB^2| + |b_2 RB^2 \cdot A^2 \times RB^2| + |b_3 RB^3 \cdot A^2 \times RB^2| \\ &= |b_1 A^2 \cdot RB^2 \times RB^1| + |b_2 A^2 \cdot RB^2 \times RB^2| + |b_3 A^2 \cdot RB^2 \times RB^3| \\ &= |-b_1 A^2 \cdot RB^3| + |b_2 A^2 \cdot 0| + |b_3 A^2 \cdot RB^1| \\ &= |-b_1 R_{23}| + |b_3 R_{21}| = b_1 |R_{23}| + b_3 |R_{21}| \end{aligned}$$

$$|T \cdot L| = |T \cdot A^2 \times RB^2| = |[T_1 \ T_2 \ T_3] \cdot [R_{32} \ 0 \ -R_{12}]] = |T_1 R_{32} - T_3 R_{12}|$$

$$|T_1 R_{32} - T_3 R_{12}| > a_1 |R_{32}| + a_3 |R_{12}| + b_1 |R_{23}| + b_3 |R_{21}| \quad A^2 \times RB^2 \quad \text{separating axis}$$

$$12) L = A^2 \times RB^3 = [0 \ 1 \ 0] \times [R_{13} \ R_{23} \ R_{33}] = [R_{33} \ 0 \ -R_{13}]$$

$$\begin{aligned} r_A |L| &= |a_1 A^1 \cdot A^2 \times RB^3| + |a_2 A^2 \cdot A^2 \times RB^3| + |a_3 A^3 \cdot A^2 \times RB^3| \\ &= |a_1 RB^3 \cdot A^1 \times A^2| + |a_2 RB^3 \cdot A^2 \times A^2| + |a_3 RB^3 \cdot A^3 \times A^2| \\ &= |a_1 RB^3 \cdot A^3| + |a_2 RB^3 \cdot 0| + |a_3 RB^3 \cdot (-A^1)| \\ &= |a_1 R_{33}| + |-a_3 R_{13}| = a_1 |R_{33}| + a_3 |R_{13}| \end{aligned}$$

$$\begin{aligned} r_B |L| &= |b_1 RB^1 \cdot A^2 \times RB^3| + |b_2 RB^2 \cdot A^2 \times RB^3| + |b_3 RB^3 \cdot A^2 \times RB^3| \\ &= |b_1 A^2 \cdot RB^3 \times RB^1| + |b_2 A^2 \cdot RB^3 \times RB^2| + |b_3 A^2 \cdot RB^3 \times RB^3| \\ &= |b_1 A^2 \cdot RB^2| + |-b_2 A^2 \cdot RB^1| + |b_3 A^2 \cdot 0| \\ &= |b_1 R_{22}| + |-b_2 R_{21}| = b_1 |R_{22}| + b_2 |R_{21}| \end{aligned}$$

$$|T \cdot L| = |T \cdot A^2 \times RB^3| = [T_1 \ T_2 \ T_3] \cdot [R_{33} \ 0 \ -R_{13}] = |T_1 R_{33} - T_3 R_{13}|$$

$$|T_1 R_{33} - T_3 R_{13}| > a_1 |R_{33}| + a_3 |R_{13}| + b_1 |R_{22}| + b_2 |R_{21}| \quad A^2 \times RB^3 \quad \text{separating axis}$$

$$13) L = A^3 \times RB^1 = [0 \ 0 \ 1] \times [R_{11} \ R_{21} \ R_{31}] = [-R_{21} \ R_{11} \ 0]$$

$$\begin{aligned} r_A |L| &= |a_1 A^1 \cdot A^3 \times RB^1| + |a_2 A^2 \cdot A^3 \times RB^1| + |a_3 A^3 \cdot A^3 \times RB^1| \\ &= |a_1 RB^1 \cdot A^1 \times A^3| + |a_2 RB^1 \cdot A^2 \times A^3| + |a_3 RB^1 \cdot A^3 \times A^3| \\ &= |-a_1 RB^1 \cdot A^2| + |a_2 RB^1 \cdot A^1| + |a_3 RB^1 \cdot 0| \\ &= |-a_1 R_{21}| + |a_2 R_{11}| = a_1 |R_{21}| + a_2 |R_{11}| \end{aligned}$$

$$\begin{aligned} r_B |L| &= |b_1 RB^1 \cdot A^3 \times RB^1| + |b_2 RB^2 \cdot A^3 \times RB^1| + |b_3 RB^3 \cdot A^3 \times RB^1| \\ &= |b_1 A^3 \cdot RB^1 \times RB^1| + |b_2 A^3 \cdot RB^1 \times RB^2| + |b_3 A^3 \cdot RB^1 \times RB^3| \\ &= |b_1 A^3 \cdot 0| + |b_2 A^3 \cdot RB^3| + |-b_3 A^3 \cdot RB^2| \\ &= |b_2 R_{33}| + |-b_3 R_{23}| = b_2 |R_{33}| + b_3 |R_{32}| \end{aligned}$$

$$|T \cdot L| = |T \cdot A^3 \times RB^1| = [T_1 \ T_2 \ T_3] \cdot [-R_{21} \ R_{11} \ 0] = |-T_1 R_{21} + T_2 R_{11}|$$

$$|-T_1 R_{21} + T_2 R_{11}| > a_1 |R_{21}| + a_2 |R_{11}| + b_2 |R_{33}| + b_3 |R_{32}| \quad A^3 \times RB^1 \quad \text{separating axis}$$

$$14) L = A^3 \times RB^2 = [0 \ 0 \ 1] \times [R_{12} \ R_{22} \ R_{32}] = [-R_{22} \ R_{12} \ 0]$$

$$\begin{aligned} r_A |L| &= |a_1 A^1 \cdot A^3 \times RB^2| + |a_2 A^2 \cdot A^3 \times RB^2| + |a_3 A^3 \cdot A^3 \times RB^2| \\ &= |a_1 RB^2 \cdot A^1 \times A^3| + |a_2 RB^2 \cdot A^2 \times A^3| + |a_3 RB^2 \cdot A^3 \times A^3| \\ &= |-a_1 RB^2 \cdot A^2| + |a_2 RB^2 \cdot A^1| + |a_3 RB^2 \cdot 0| \\ &= |-a_1 R_{22}| + |a_2 R_{12}| = a_1 |R_{22}| + a_2 |R_{12}| \end{aligned}$$

$$\begin{aligned} r_B |L| &= |b_1 RB^1 \cdot A^3 \times RB^2| + |b_2 RB^2 \cdot A^3 \times RB^2| + |b_3 RB^3 \cdot A^3 \times RB^2| \\ &= |b_1 A^3 \cdot RB^2 \times RB^1| + |b_2 A^3 \cdot RB^2 \times RB^2| + |b_3 A^3 \cdot RB^2 \times RB^3| \\ &= |-b_1 A^3 \cdot RB^3| + |b_2 A^3 \cdot 0| + |b_3 A^3 \cdot RB^1| \\ &= |-b_1 R_{33}| + |b_3 R_{13}| = b_1 |R_{33}| + b_3 |R_{13}| \end{aligned}$$

$$|T \cdot L| = |T \cdot A^3 \times RB^2| = [T_1 \ T_2 \ T_3] \cdot [-R_{22} \ R_{12} \ 0] = |-T_1 R_{22} + T_2 R_{12}|$$

$$|-T_1 R_{22} + T_2 R_{12}| > a_1 |R_{22}| + a_2 |R_{12}| + b_1 |R_{33}| + b_3 |R_{13}| \quad A^3 \times RB^2 \quad \text{separating axis}$$

$$15) L = A^3 \times RB^3 = [0 \ 0 \ 1] \times [R_{13} \ R_{23} \ R_{33}] = [-R_{23} \ R_{13} \ 0]$$

$$\begin{aligned} r_A |L| &= |a_1 A^1 \cdot A^3 \times RB^3| + |a_2 A^2 \cdot A^3 \times RB^3| + |a_3 A^3 \cdot A^3 \times RB^3| \\ &= |a_1 RB^3 \cdot A^1 \times A^3| + |a_2 RB^3 \cdot A^2 \times A^3| + |a_3 RB^3 \cdot A^3 \times A^3| \\ &= |-a_1 RB^3 \cdot A^2| + |a_2 RB^3 \cdot A^1| + |a_3 RB^3 \cdot 0| \\ &= |-a_1 R_{23}| + |a_2 R_{13}| = a_1 |R_{23}| + a_2 |R_{13}| \end{aligned}$$

$$\begin{aligned} r_B |L| &= |b_1 RB^1 \cdot A^3 \times RB^3| + |b_2 RB^2 \cdot A^3 \times RB^3| + |b_3 RB^3 \cdot A^3 \times RB^3| \\ &= |b_1 A^3 \cdot RB^3 \times RB^1| + |b_2 A^3 \cdot RB^3 \times RB^2| + |b_3 A^3 \cdot RB^3 \times RB^3| \\ &= |b_1 A^3 \cdot RB^2| + |-b_2 A^3 \cdot RB^1| + |-b_3 A^3 \cdot 0| \\ &= |b_1 R_{32}| + |-b_2 R_{31}| = b_1 |R_{32}| + b_2 |R_{31}| \end{aligned}$$

$$|T \cdot L| = |T \cdot A^3 \times RB^3| = [T_1 \ T_2 \ T_3] \cdot [-R_{23} \ R_{13} \ 0] = |-T_1 R_{23} + T_2 R_{13}|$$

$$|-T_1 R_{23} + T_2 R_{13}| > a_1 |R_{23}| + a_2 |R_{13}| + b_1 |R_{32}| + b_2 |R_{31}| \quad A^3 \times RB^1 \quad \text{separating axis}$$